Figure 2 presents a similar set of plots corresponding to Fig. 1, with the exception that $\omega = 0.05$, which corresponds to the second data set shown in Table 1. Again, the proposed procedure works well, even in the presence of noise. A favorable comparison is evident. The radiosity results when $N = 4$ produced by the method of Shih et al.[1] is shown in Fig. 2a. The radiosity distribution as developed by the proposed method when $N = 6$ is displayed in Fig. 2c. Likewise, the resulting temperature distribution shown in Fig. 2b, when $N = 4$, is clearly in line with the results offered in Fig. 2d when $N = 6$.

To conclude this Note, the proposed methodology possesses merit in situations that do not easily permit the development of an exact solution. This approach deserves more rigorous development and attention.

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Heat Transfer Transients in Stagnation Flows Due to Changes in Flow Velocity

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Nomenclature

C = steady-state freestream velocity gradient
C_* = constant dimensionless freestream velocity gradient from potential flow theory, Cw/V_i
h = heat transfer coefficient
k = thermal conductivity
Nu_* = ratio of instantaneous Nusselt number to steady-state Nusselt number, Nu_w/Nu_{w0}
t = time
U_∞ = local velocity component parallel to the surface in the freestream
$U_{\infty*}$ = dimensionless freestream velocity, U_∞/V_{i0}
V_i = incident flow velocity
w = characteristic length scale, e.g., jet width or cylinder diameter
x = distance along impingement surface from stagnation line
Γ = dimensionless thermal boundary-layer thickness, $C\Delta^2/\nu$
Λ = dimensionless hydrodynamic boundary-layer thickness, $C\delta^2/\nu$
ν = kinematic viscosity
τ = dimensionless time, Ct

Subscript
0 = pertaining to $t = 0$ and steady-state conditions

Introduction

CONVECTIVE heat transfer often occurs where flow or surface-related disturbances are intentionally induced or are the result of some change in operating conditions. Such disturbances induce thermal transients in the fluid that propagate within the fluid and cause changes in heat transfer coefficients. Examples of disturbances include pulsations in an impinging jet flow, velocity changes due to incident large-scale flow structures, and changed heat fluxes at a surface. An understanding and quantification of the ensuing thermal transients can be useful in assessing whether available heat transfer correlations can be implemented in developing simple thermal models or in efforts to employ nonlinear dynamical effects to induce changes in time-averaged heat transfer coefficients. Recently, e.g., theoretical models have been developed to disclose over long time intervals nonlinear dynamical effects in stagnation flows.[1,2] With nonlinear dynamical effects included, sinusoidal fluctuations can lead to nonsinusoidal responses in boundary layers, and thereby, reveal conditions where time-averaged heat transfer can be altered.

In this study, a model[2] of instantaneous convective heat transfer in a planar stagnation flow was implemented to determine specifically responses to single ramp-up or ramp-down changes in the incident flow velocity. A primary motivation for this work was the need for documentation of transient effects in a simplified format with the realization that implementation of the recently developed nonlinear dynamical model is difficult in practice. Response times for convective heat transfer coefficients are given in a generalized format to account for dependencies on flow velocities, characteristic dimensions such as nozzle widths or cylinder diameters, and thermophysical properties. Equivalent first-order time constants and transient durations are tabulated for ready use. A systematic presentation of the nonlinear dynamical transients of this study is available in conference proceedings.[3]

Analytical Methods

Complete documentation of the nonlinear dynamical model that is implemented here has been published previously.[2] In summary, a solution methodology was sought that was consistent with approaches used in recent studies of nonlinear dynamics and chaos in thermal or discrete mechanical systems. The responses of such systems are commonly represented by a system of first-order, ordinary differential equations (ODEs). The approach that was selected is related to the von Kármán–Pohlhausen technique,[4] since this technique has been widely used in studies of steady stagnation flows and

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can be implemented in a manner such that nonlinear terms are retained. In the related technique, fourth-order, temporally adaptive profiles were used for fluid velocity and temperature to obtain governing first-order differential equations for the hydrodynamic and thermal boundary-layer responses from the partial differential and integral equations for momentum and energy conservation. Temporal adaptivity allowed the profiles to independently change shape in response to instantaneous conditions. Fluctuations in the incident velocity components led to temporal variations in the hydrodynamic boundary layer δ, the thermal boundary layer Δ, as well as the surface temperature T_s. Symmetry in the dividing flow yielded a similar symmetry in the boundary layers with respect to the plane $x = 0$ and conveniently removed any spatial dependencies from model equations. The local heat transfer coefficient was therefore spatially constant in the vicinity of the stagnation line, and so results of this study can be extended to nearby regions.

The model was developed under the following general assumptions: 1) incompressible laminar flow, 2) constant thermophysical properties, 3) negligible viscous heating, 4) negligible body forces in comparison to viscous forces, and 5) constant freestream temperature T_∞. Governing equations were nondimensionalized to minimize the number of independent parameters and thereby add generality to the results. The hydrodynamic and thermal boundary-layer thicknesses were represented by the dimensionless variables Λ and Γ, respectively, and time was given by the variable τ $(=Ct)$. A nonlinear system of three ODEs was obtained for the boundary-layer responses. The nonlinear differential equations for Λ, Γ, and the dimensionless surface temperature θ_s $[=(T_s - T_\infty)/(T_{s0} - T_\infty)]$ are given in a prior paper.[2]

The relation between θ_s and the dimensionless surface heat flux q_{s*} is given in Eq. (1). Since in this study a constant surface heat flux q_s was specified and transients began from steady-state conditions $[\mathrm{d}\theta_s/\mathrm{d}\tau = 0$ in Eq. (1a)], $q_{s*} = 2\Gamma_0^{-1/2}$:

$$\frac{\mathrm{d}\theta_s}{\mathrm{d}\tau} = \frac{6}{Pr\Gamma}(q_{s*}\Gamma^{1/2} - 2\theta_s) \tag{1a}$$

$$q_{s*} = \frac{q_s}{k\sqrt{(C/\nu)}(T_{s0} - T_\infty)} \tag{1b}$$

Although this work has been performed for a constant surface heat flux, transients of Ref. 1 suggest that similar timescales pertain to surfaces with a constant surface temperature.

Instantaneous Nusselt numbers referenced to a characteristic dimension w for the planar stagnation flow were determined from Newton's law of cooling and the temporally adaptive temperature profiles in the fluid. The resulting expression is given by Eq. (2):

$$Nu_w = \frac{wq_{s*}}{\theta_s}\sqrt{\frac{C}{\nu}} \tag{2}$$

The single ramp-up or ramp-down variations in the incident flow velocity were modeled as portions of sine waves in order that variations would be realistically smooth. The mathematical formulation is presented in the following equations:

$$0 < \tau < \tau_D{:}\quad U_{\infty*} = C_* x_* \left\{1 + \frac{\varepsilon}{2}\left[1 + \sin\left(\frac{\pi\tau}{\tau_D} - \frac{\pi}{2}\right)\right]\right\} \tag{3a}$$

$$\tau > \tau_D{:}\quad U_{\infty*} = C_* x_*(1 + \varepsilon) \tag{3b}$$

Changes in incident flow velocity thereby corresponded to the portion of a sine wave extending from $-\pi/2$ to $\pi/2$ rad. Changes occurred more rapidly for shorter duration times as specified by the parameters τ_D. For the purpose of discussion the steady velocities preceding and following the ramp changes will be referred to as end-point velocities.

Steady-state values for Λ_0, Γ_0, and q_{s0*} were determined from the system of ODEs for Λ, Γ, and θ_s with all time-derivatives set to zero. The secant method[5] was utilized to solve the resulting transcendental equations to within a convergence tolerance of 10^{-6}. The fourth-order Runge–Kutta algorithm was implemented to obtain numerical solutions. Time steps were controlled and successively halved until a tolerance of 10^{-4} was achieved between successive calculations. The model can become unstable under a combination of high-amplitude and hf fluctuations in the forcing functions.[6] Verification of the modified model was provided by comparison to results obtained previously with the original model. Calculated transients were identical. The original model was previously verified against independent experimental and theoretical results for periodic stagnation flows.

Results and Discussion

The nonlinear system of ODEs[2,6] and the ramp variation in flow velocity given by Eq. (3) include as parameters the Prandtl number Pr, the flow velocity disturbance duration τ_D, and the fractional change ε in the incident flow velocity. This small number of parameters was made possible by a careful selection of the dimensionless variables. The dimensionless time τ $(=Ct)$, as well as the dimensionless boundary-layer thicknesses Λ and Γ, are related to the velocity gradient $C(U_\infty = Cx)$ at the stagnation line for steady-flow conditions. Freestream velocity gradients are smaller in stagnation flows with larger characteristic dimensions. The hydrodynamic and thermal boundary-layer responses are thereby less rapid for impinging jets with larger widths or for cylinders, turbine blades, or sensor bodies with larger characteristic sizes. Applicable velocity gradients for specific stagnation flows may be used to convert the dimensionless results of this study to dimensional values. For example, $C_* = 4.0$ for a cylinder in a crossflow, $C_* = 0.785$ for a uniform planar jet flow, and $C_* \approx 1.73$ for a planar jet flow with an incident parabolic velocity profile.[1]

The influences of the rapidity of a ramp decrease in incident flow velocity on Nusselt number responses are shown in Fig. 1 for $\varepsilon = -0.4$. The Nusselt number responses are normalized by the initial steady-state Nusselt number to show effects most clearly. Since the boundary-layer dynamics reflected momentum and energy transfers within the fluid, the surface tem-

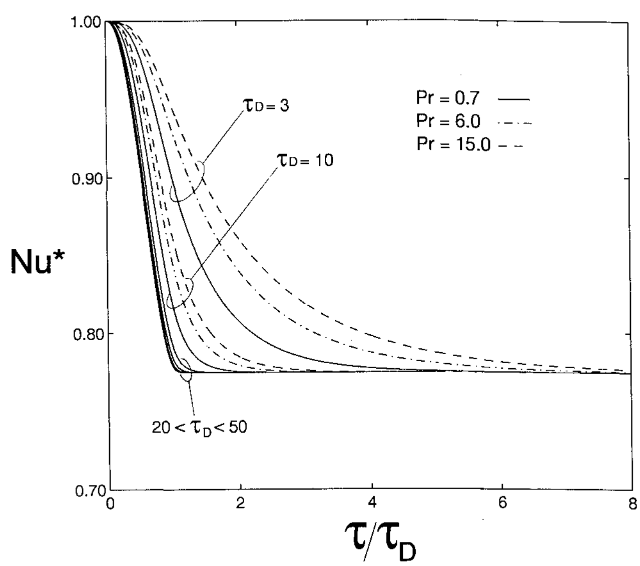

Fig. 1 Model-predicted transients in Nusselt number due to a decrease ($\varepsilon = -0.4$) in incident flow velocity for various disturbance durations and Prandtl numbers.

Table 1 Summary of transient durations and equivalent first-order time constants for Nusselt number

		$Pr = 0.7$		$Pr = 3.0$		$Pr = 15.0$	
τ_D	ε	τ_1	τ_2	τ_1	τ_2	τ_1	τ_2
3	−0.8	5.52	28.86	6.84	37.89	9.66	56.64
3	−0.6	4.44	16.83	5.37	21.75	7.35	32.22
3	−0.4	3.93	12.42	4.65	15.93	6.21	23.22
3	−0.2	3.60	10.08	4.20	12.81	5.49	18.51
3	0.2	3.18	7.62	3.66	9.51	4.65	13.50
3	0.8	2.88	5.88	3.21	7.20	3.96	9.96
10	−0.8	9.80	33.30	10.95	42.15	13.55	60.70
10	−0.6	8.80	21.35	9.55	26.15	11.30	36.30
10	−0.4	8.20	17.05	8.85	20.30	10.25	27.35
10	−0.2	7.85	14.80	8.40	17.25	9.60	22.65
10	0.2	7.35	12.50	7.80	14.15	8.75	17.75
10	0.8	6.85	11.05	7.25	12.05	8.05	14.40
50	−0.8	36.00	64.70	37.10	72.30	39.45	88.80
50	−0.6	33.80	54.15	34.65	57.65	36.45	65.75
50	−0.4	32.50	50.95	33.25	52.90	34.75	57.85
50	−0.2	31.65	49.60	32.25	50.80	33.55	54.05
50	0.2	30.45	48.40	30.95	49.05	32.00	50.70
50	0.8	29.30	47.65	29.70	48.05	30.50	48.95

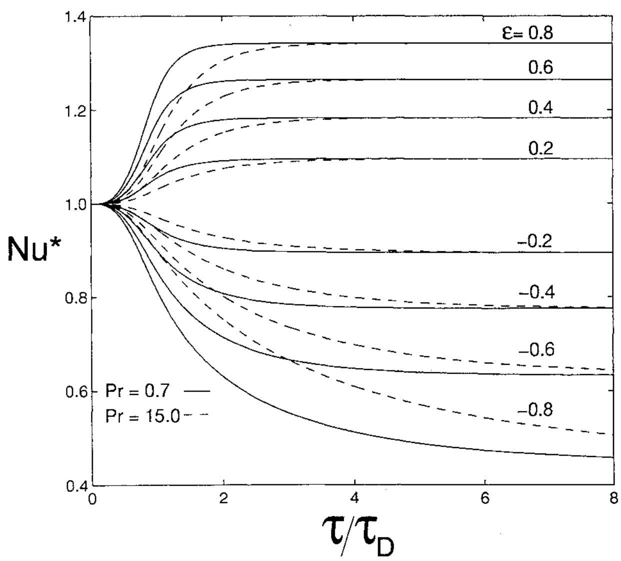

Fig. 2 Nusselt number variations due to increases or decreases in incident flow velocity for $\tau_D = 3$.

perature and Nusselt number responses in Fig. 1 lagged the boundary-layer responses. The corresponding hydrodynamic and thermal boundary-layer thicknesses increased monotonically due to the decrease in flow velocity.[3] For $\tau_D > 20$, transients were sufficiently slow to allow quasisteady behavior for which Nu_w was accurately described by available heat transfer correlations and the instantaneous flow velocity. (Quasisteady behavior for Nu_w was indicated by the overlapping curves at the larger τ_D.) This behavior did not occur for shorter disturbance durations since momentum and energy transports within the boundary layers were not sufficiently fast to maintain equilibrium conditions. For $Pr = 15$ and $\tau_D = 3$ in Fig. 1, e.g., transients persisted appreciably longer than the disturbance duration τ_D. Interestingly, τ_D most strongly dictated whether quasisteady behavior occurred; Pr had a lesser influence. The Γ and Nu_w responses were found to be more rapid for smaller Pr, owing to higher thermal diffusivities.

The influence of ε in the incident flow velocity [Eq. (3)] is shown in Fig. 2. The durations of the transients were longer for the ramp-down changes ($\varepsilon < 0$) in the incident flow velocity. Moreover, transient durations for the ramp-up changes in incident flow velocity decreased monotonically as ε increased. The shorter transient times for ramp-up changes were attributable to increased momentum and energy advection within the boundary layers due to the increased flow velocity. For example, steady-state Nusselt numbers were re-established more quickly for $\varepsilon = 0.8$ in comparison to $\varepsilon = 0.2$. Transients for $Pr = 0.7$ were more rapid in comparison to those for $Pr = 15$, owing to a higher thermal diffusivity in the fluid. Otherwise, all transients are qualitatively similar. However, it should be noted that transient durations for both increases and decreases in flow velocity were found to be nearly identical when the transients occurred between the same endpoint velocities [Eq. (3)]. Although the ramp variations are specified in a familiar form, Eq. (3) returns different endpoint velocities for ramp-up ($\varepsilon > 0$) and ramp-down ($\varepsilon < 0$) changes for the same ε.

While each transient pertains to specific parametric conditions in the nonlinear system of ODEs, responses resemble those of first-order linear systems to a step change. Representation of the nonlinear dynamics by an equivalent first-order system therefore seemed reasonable. Equivalent first-order time constants given by τ_1 that satisfy Eq. (4) were determined from the calculated responses for $0.7 < Pr < 15.0$ and $-0.8 < \varepsilon < 0.8$. In Eq. (4), $Nu_{w,f}$ and $Nu_{w,i}$ are the final and initial Nusselt numbers prior to and following a transient. Each equivalent first-order time constant τ_1 was taken to be the value of τ where $(Nu_w - Nu_{w,i})/(Nu_{w,f} - Nu_{w,i}) = 1 - e^{-1}$. Equation (4) returns instantaneous values of Nu_w to within 14% of model calculations for $\tau_D < 20$:

$$Nu_w = (Nu_{w,f} - Nu_{w,i})(1 - e^{-\tau/\tau_1}) + Nu_{w,i} \tag{4}$$

The durations of transients in the Nusselt number were also determined from calculated responses. These durations were defined to be the elapsed times needed to reach the new steady-state Nusselt number $Nu_{w,f}$ to within 1% and are given by τ_2. Results are summarized in Table 1. Linear interpolation within table will yield values to within 4.5% of model-predicted values. The results of this study can be used to assess whether available heat transfer correlations, which are intended for steady-state, are applicable to a given transient condition. Applicability is suggested when $\tau_D \gg \tau_1$. Equivalent first-order time constants can be used to develop simplified thermal expressions of transient responses. For example, such expressions can be included in control models of thermal sensors.

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